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In all probability an isotope will decay over a given period of time. The cost of car insurance depends on the probability that the driver will be involved in accidents. Even gamblers use probabilities as they contemplate their next move. Probabilities are everywhere and permeate our lives. But although probability is seemingly ubiquitous, it is not so easy to define or to explain its philosophical foundations. Timothy Childer's *Philosophy and Probability* does just that by introducing several leading interpretations of probability, and giving an overview of the philosophical issues they raise. The author also provides an appendix, explaining the basic mathematics needed to handle the probability calculi.

The first chapter is concerned with the frequentist interpretation of probability. Frequentism requires mass phenomena, i.e. experiments that can be (ideally) repeated an unlimited number of times. As an example, think of a coin being endlessly tossed. Now, let us construct the sample space, viz., the set of all infinite sequences whose only members are the two possible outcomes of a tossed coin (heads and tails). A collective is a member of the sample space. Suppose $n(H)$ is the number of heads in n tosses in a given collective. The relative frequency of heads in n tosses is $n(H)/n$. The frequentist probability of heads is its limiting relative frequency, i.e. the value $n(H)/n$ takes as n approaches infinity. Von Mises, one of the greatest frequentists of the 20^{th} century, integrated the frequentist theory with two axioms. He introduced the *Axiom of Convergence*: for a given collective, the limiting relative frequency does exist (p.6). This means that the limiting relative frequency of a collective does not oscillate, but settles down to some definite value. Von Mises also formulated the *Randomness Axiom*, according to which arbitrarily chosen subsequences of a collective have the same limiting frequency as the collective itself. Von Mises' notion of randomness set off a vigorous debate, involving logicians such as Alonzo Church and Andrey Kolmogorov (pp.10-14). Moreover, frequentism plays an important role relative as far as the Kolmogorov axioms are concerned. These axioms conceive probability as measuring how events are independent from one another. They state that (i) the probability of *A* is between 0 and 1, and (ii) the probability of any finite collection of mutually exclusive events adds. These axioms can be rewritten in terms of their limiting relative frequencies. Hence, how *A* and *B* are independent can be explained in terms of *A*'s and *B*'s limiting relative frequencies. Frequentism, however, leads to some specific difficulties. Limiting frequencies are mathematical limits, and they seem to lack a link with accessible evidence (pp.20-22). Furthermore, frequentism cannot predict single-case probabilities, because limiting frequencies only attach to (infinite) classes of outcomes (pp.22-24).

The second chapter deals with the propensity interpretation. According to this view, the probabilities invoked by axioms (i)-(ii) are not properties of classes of outcomes, but things characterising individual events. Thus, the probability that a single flipped coin will land heads is interpreted as a disposition. A disposition is, in turn, a relation that a single outcome bears to its generating conditions. Although at first sight, the propensity interpretation seems to capture the intended meaning of several physical laws, it nevertheless faces numerous problems. The first one is epistemological. How do we isolate the right generating conditions? It might seem tempting to say that the generating conditions have to be identified either with the total state of the universe at a specific time, or with the light cone of a space-time point. But since we cannot know things such as the total state of the universe, the propensity interpretation may appear to lack empirical content. On the other hand, if the generating conditions are local sets of constraints, an ontological problem can arise. If the universe obeys determinism, there are no non-trivial probabilities. Hence, the complaint that "this propensity approach [...] must prejudge the question of determinism in favor of indeterminism" (p.38). The propensity approach has also been accused of uncovering several shortcomings when applied to interpret the probability calculus. Probability measures the propensity for something to occur, provided that there are certain conditions. Thus, the propensity approach is in some way related to causality. Suppose when the weather is hot, Bob's propensity to drink beer is of 0.95. Assume that 0.6 is the propensity for the weather to be hot, and that Bob's propensity to drink beer is 0.7. According to Bayes' theorem, the weather's propensity to be hot, given that Bob

drinks beer, is 0.81. But this is absurd, for how could Bob's behavior have any causal influence on the weather? (p.41)
Both the frequentist and the propensity approaches treat probabilities as objective features of reality. Nevertheless, probabilities can also model degrees of belief. This latter view, known as Bayesianism, and it is the subject matter of chapter three. The core idea of this subjective interpretation is that the more strongly you believe in *q*, the more you will be willing to lose by betting on *q*. This approach is based, of course, on the notion of a fair bet. A bet is fair if one party does not necessarily loose. For instance, "If the coin lands heads, I win; if it lands tails, you loose" is not a fair bet. The Ramsey-De Finetti theorem establishes that all and only fair bets obey axioms (i)-(ii). Let us define the odds of a bet as b/a, where a is the sum someone is prepared to win, and b is the sum someone is willing to loose. The betting quotient p is $(b/a)/(1+b/a)$, and the total amount of money at stake is $S=a+b$. The quotient of a fair bet on A, say $p(A)$, must lie between 0 and 1. Indeed, if $p(A)<0$, a is positive and b is negative, regardless if A turns out to be true or false. This means that the bet cannot be fair, since one of the opponents cannot loose. On the other hand, if A is necessarily true, let us consider $a=S(1-p(A))$. Now, $p(A)<1$ is impossible, and if $p(A)$ were grater than 1, then the person betting on A would surely win. Hence, the only fair bets on necessarily true propositions are such that their quotient p equals 1. It follows that $0\leq p(A)\leq 1$. A little algebra shows that the quotient $p(A \vee B)$ adds (that is, $p(A \vee B)=p(A)+p(B)$), if A and B are two mutually exclusive propositions. It follows that p satisfies axioms (i)-(ii) of the probability calculus, and can be taken as a measure of belief (p.58). Bayesianism has several applications in epistemology, mostly due to Bayes' theorem. The theorem establishes that the probability of the hypothesis *h*, given evidence *e*, equals the product of the probability of *h* and the degree to which *h* predicts *e* over the probability of *e*. The theorem accounts for evidence affecting beliefs, and it yields a solution to the so-called Duhem-Quine problem (p.68-75). Many philosophers have nevertheless questioned the principle that "people are willing to offer the same betting quotients singly that they would offer jointly", which is needed for a betting quotient to satisfy (ii). Bayesianism has also been charged of omitting many qualitative features that are relevant for

epistemology, for “it does not distinguish between novel predictions and *ad hoc* adjustments in a theory” (p.98).
Is there any relation between objective and subjective interpretations? Chapter four explains the strategies philosophers have developed to characterize this connection. On the one hand, Monists claim that there are subjective probabilities only, and that the objective explanation of probability must be translated into a subjective one. This strategy has its difficulties, as an analysis of De Finetti’s method shows (pp.108-111). On the other hand, Dualists hold that there are both subjective and objective probabilities, and “they can be combined by letting values of objective probabilities serve as evidence in Bayesian calculations” (p.101). One of the most influential theories for Dualists is David Lewis’ *Principal Principle*: the objective probability of *A* in world *w* at time *t* equals the subjective probability of *A*, conditioned on the theory of chance for world *w*, on the history of *w* up to *t*. A theory of chance for *w* specifies the objective probabilities of an outcome at *w* up to *t*. Lewis’ principle is compatible with both Bayesianism and the propensity interpretation, but it fails to be metaphysically neutral. The author shows that the Principal Principle is *prima facie* at odds with a humean approach towards supervenience and natural laws. Furthermore, Childers goes on to illustrate some strategies humeans may adopt to address these issues (pp.104-108).
Chapter five treats the classical and logical interpretations. The former approach addresses the problem of how to distribute winnings when a game is interrupted before its expected end. The key principle here is that of *Indifference*: the probability of an event is the number of events favorable to that event, divided by the total number of possibilities (p.114). The principle called the *Rule of Succession*, $(n+1)/(m+2)$, is the probability that an event will be repeated, given *n* previous observations of the event out of *m* observations (p.116). The *Rule of Succession* is an easy way out for the problem of induction, while the *Principle of Indifference* leads to paradoxes (p.119-123). The logical interpretation is an updated version of the classical one, and it takes ‘probability’ to mean ‘partial entailment’. Rudolf Carnap’s inductive logic (pp.127-132) assumes that the range of a sentence *q* to be the set of maximal consistent sets of sentences containing *q*. Probability is then taken to measure ranges. Carnap defines a function *m* such that (iii) $\Sigma_i m(P_i)=1$ for any

maximal consistent set P_i, (iv) if q is logically possible, $m(q)=\Sigma_j m(P_j)$, where the P_j's are in the range of q, and (v) if q is logically false, $m(q)=0$. Moreover, the measure of how e partially entails h is $m(e\wedge h)/m(e)$. This inductive logic generalizes some aspects of the classical approach, but it has no straightforward generalization for continuum probability assignments. This restriction, however, makes Carnap's method unsuitable for science (p.132).

The sixth chapter introduces the information-based approach. A unit (bit) of information represents a system existing in one state instead of in another (for instance, a switch being on instead of being off). Bits of information are thus modeled as (indicator variable) functions, ranging over {1,0}. Since n indicator variables can describe 2^n states, the natural measure for an increase in descriptive power is logarithmic. For instance, given n potential states, the amount of information they convey is $log_2 n$. The core idea, then, is to relate probability to certainty, and certainty to information. The more certain (probable) A is, the less information it carries (in symbols, $I(A)=-log_2 p(A)$). If entropy is taken to measure where the probability of A concentrates, entropy would be at its maximum when the probability of A and that of $\neg A$ are equal. This relation may suggest the adoption of the *Principle of Maximum Entropy*: when no information on the values that a set of variables takes is available, probabilities should maximize entropy (p.138). This principle is an information-based formulation of the *Principle of Indifference*, but it yields shortcomings that are quite close to those of the classical and the logical interpretations (p.140-151).

To conclude then, Childer's book offers a detailed overview of the contemporary debate on the philosophy of probability, and provides a critical analysis of several of its leading interpretations. It is an excellent guide to all those who are intrigued by the foundations of probability calculi.