Dorota Leszczyńska-Jasion, *The Method of Socratic Proofs*, Springer, Cham 2025, pp. 348, € 145.49, ISBN 9783031824500

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The Method of Socratic Proofs by Dorota Leszczyńska-Jasion is a technically focused systematic presentation of the proof theory of erotetic calculi, formal proof systems that transform questions into other questions. After introducing the conceptual context from which these systems originate, the book proceeds to study calculi for propositional and first-order classical logic, standard modal logics and intuitionistic logic, the latter being a novel addition to the framework. Then, it investigates their connection with the underlying intuitions and with standard Gentzen-style sequent calculi for these logics.

Chapter 1 introduces the conceptual framework at the basis of erotetic calculi and positions the book with respect to the literature on the logic of questions, referring both to inferential erotetic logic, in which the book's results are situated, and to other formal approaches to questions. Two crucial notions are introduced: *erotetic inference*, a relation that holds between two questions when answers to the second one bring one closer to resolving the first (modelling reasonable steps in the process of inquiry); and the notion of *erotetic calculi*, proof systems that transform questions into other questions in accordance with erotetic inference. Series of valid steps in an erotetic calculus are called *socratic transformations*. Finally, general research goals for logics of questions are described to provide orientation for the later chapters. Most of the book's conceptual and informal material is encompassed by this chapter, which, additionally, provides a particularly rich set of references for the numerous topics and research areas discussed.

In Chapter 2, erotetic calculi are given for propositional and first-order classical logic. After concisely providing the necessary background for these underlying logics, the author introduces the formalism and notation used for the calculi throughout the book. Six calculi are then presented, covering three variants for both the propositional and the first-order case. The calculi are based on sequents, which, intuitively, represent entailment claims between two sets of formulas, called *antecedent* and *conclusion*. The three variants differ in the form of the sequents: one with an arbitrary antecedent and a single-formula conclusion, one with an arbitrary antecedent and an empty conclusion, and one that is multi-conclusioned but has an empty antecedent. For each variant, an intuitive interpretation is given; these interpretations are then expanded upon in Chapter 5. Each line of a proof in an erotetic calculus consists of a set of sequents, which, intuitively, represents a question about the entailment claims expressed by the sequents. The remaining sections of the chapter are dedicated to describing and proving key results for the six calculi, such as soundness, completeness and their structural properties. For these calculi, the definitions of these notions and their proof strategies differ from what is standard in other fields of proof theory, and these differences are clearly highlighted and justified. The proofs of these results are highly detailed, including those parts that follow standard arguments, making them accessible even for non-specialists. Finally, a proof-search strategy is presented for the first-order case.

Chapter 3 is dedicated to developing erotetic calculi for standard propositional modal logics and proving their soundness and completeness. The structure mirrors that of Chapter 2. Here, the author only considers one sequent variant, using indices to encode the possible worlds of the underlying Kripke semantics, and having multiple non-indexed formulas in the antecedent with multiple, possibly indexed, formulas in the conclusion. The first sections introduce the basic propositional modal logic and its extensions (defined by the standard frame conditions of seriality, reflexivity, symmetry, transitivity and Euclideanness), describe the sequents and provide a calculus for each modal logic. The majority of the chapter is then dedicated to proving the soundness, completeness, and structural properties of each calculus. Completeness is proved separately for the basic case and for the extensions; proofs of the latter only detail the differences from the base case.

Chapter 4 presents a novel result, an erotetic calculus for intuitionistic propositional logic (IPC). The calculus uses sequents with multiple indexed antecedents and conclusions. After briefly introducing IPC using its Kripke semantics, the sequents and the calculus are defined. The remaining sections of the chapter are then dedicated to proving the soundness, the structural properties and the completeness of the calculus with respect to IPC semantics. The structure and the methods used are largely parallel to the modal cases, thanks to the use of Kripke semantics. For this reason, proofs are only provided for those parts where the two diverge. Nonetheless, the structure of the proofs and the intermediate results needed to achieve them are made explicit and clearly stated, while also providing references to their modal counterparts. The completeness proof in particular requires a significant adaptation compared with the modal case.

Chapter 5 revisits the conceptual basis introduced in Chapter 1, and examines it formally in light of the preceding chapters. First, erotetic implication is defined formally in the context of questions involving formulas (unlike those of the preceding chapters, which concerned sequents). Then, alternative related notions are explored, and a result is obtained about the possibility of constructing, given two questions, a third question that acts as an "interpolant" with respect to erotetic implication. Then, the focus shifts to sequents, and the rules for each of the calculi introduced in the previous chapters are shown to preserve a version of erotetic implication generalized to questions involving sequents. The last section comes back to the classical cases and proposes a notion of multi-conclusion consequence, connecting it with multi-conclusion entailment, and formally confirming the proposed interpretations of the three variants of classical sequents.

In Chapter 6, the derivations in erotetic calculi for classical logics are translated into standard Gentzen-style sequent calculi. The chapter deals with the extraction of trees from erotetic derivations, accompanying the formal exposition with numerous examples, and provides a sequent calculus for classical socratic transformations, showing how to translate a derivation in the former into a derivation in the latter. A final section briefly touches on the topic of reversed sequents, connected with refutability, and a system for socratic refutations.

Chapter 7 extends the scope of the previous chapter to the modal and intuitionistic cases. Slightly modified (but equivalent) versions of the modal and intuitionistic erotetic calculi are translated into standard Gentzen-style sequent calculi. The modal cases, restricted to non-symmetric and non-

Euclidean logics for technical reasons, are presented one by one, starting from the template of the basic modal logic and moving on to consider each extension separately. The structure is similar to the one for the classical version and algorithms for the translations are provided and proven to be correct. The final section covers the intuitionistic case in an analogous fashion.

Overall, *The Method of Socratic Proofs* is a well-structured and clearly presented systematic treatment of results and techniques concerning the proof theory of erotetic calculi, including known results as well as novel contributions. The material is clearly organized and the book successfully achieves its stated objectives, resulting in a useful reference work for readers interested in studying, adapting, or applying the formal proof-theoretic methods developed in the field of erotetic calculi. The range of proof systems presented within a unified framework effectively shows the flexibility and generality of the approach.

As for accessibility, the book makes an explicit effort to be self-contained, assuming no prior background and introducing all logical systems under consideration, while also providing references to more comprehensive treatments. Nevertheless, the technical level of the material (especially in the later chapters, primarily directed at specialists) and the (justified) conciseness of the background sections make the book more suitable for audiences already familiar with these logics and, ideally, with sequent calculi. This is in line with the author's stated aims. The first chapter, with its overview of the conceptual setting, the related literature, and the motivations underlying erotetic calculi, greatly improves accessibility and adequately introduces the reader to the field of erotetic calculi. In contrast, outside of this chapter, intuitive interpretations and conceptual discussions play a more limited role, and are only expanded upon in Chapter 5, where they are still mostly treated from a formal perspective. This approach is effective in supporting conciseness and structural clarity, but devoting slightly more space to conceptual guidance could further improve accessibility for non-specialist readers without excessively compromising the balance.

The advantages of this concise and technically focused approach are evident in the navigability of the material, whose structure is clear and easy to follow. The exposition also deals very effectively with potentially repetitive proofs, managing to avoid redundancy, while making it extremely straightforward to reconstruct omitted arguments through detailed parallels with previous proofs and explicit statements of the required lemmas and propositions, and spelling out in detail those parts of the arguments that require adaptation to the specific setting.

Taken together, these features suggest that the book is best approached as a technical reference, of particular value to readers interested in the proof-theoretic development and application of erotetic calculi.