Aaron Cotnoir, Achille Varzi, *Mereology*, Oxford University Press, Oxford 2021, pp. 405, $ 105.00, ISBN 9780198749004

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Among the topics that have always attracted attention in philosophical investigation, one finds the relationship between an entity and its parts and between the different parts that compose it. Mereology is that branch of philosophical knowledge that deals with parts and part-whole relationships. Aaron Cotnoir and Achille Varzi's *Mereology* is essential for those interested in formal mereology, its historical-conceptual journey, and its metaphysical debates.

Chapter 1 is primarily devoted to a historical-conceptual survey of the origins and developments of mereology, ranging from the Presocratics to contemporary authors. In Chapters 2-5, Cotnoir and Varzi provide a detailed presentation of Classical Mereology (CM) and discuss its underlying principles. The final chapter illustrates various non-classical mereologies, some of which are introduced to address specific metaphysical problems. Overall, these non-classical theories either weaken or strengthen some of the principles of CM, either strictly logical or mereological.

In Chapter 1, Cotnoir and Varzi trace the main historical stages that have led mereology to assume a significant philosophical prominence to become a formal domain of inquiry. Excluding a brief untitled text dating back to around 1690 by Leibniz, in which the mereological containment relation is introduced, the earliest preludes of a full-fledged mereological theory are found in Brentano's *Kategorienlehre* and Husserl's *Logische Untersuchungen*. However, it is through Leśniewski's *Podstawy ogólnej teoryi mnogości* (1916) and Leonard and Goodman's *The Calculus of Individuals and its Uses* (1940) that CM received its first rigorous formalization, albeit in different ways. Remarkably, although some authors, such as Aristotle or Husserl, investigated the parthood relation "x is part of y" presupposing that 'y' constitutes a whole (holology), mereology itself is not committed to this assumption. In fact, "being a whole" is a predicate that goes beyond the expressive power of

mereology, which is bounded to the predicates of parthood and identity.

In Chapter 2, Cotnoir and Varzi assume that the part-whole relation is a partial order (i.e., reflexive, antisymmetric, and transitive). To formalize CM, additional axioms are required. These axioms are either principles of decomposition or principles of composition. Roughly, the former governs the possibility of passing from a whole to its constituent parts while the latter does the opposite. Not all formulations of CM share the same principles of decomposition and composition. To formalize CM Cotnoir and Varzi present the tenets of Remainder (R) and Unrestricted Composition (UC). R is a decomposition principle and states that if x is not part of y, then there exists something, z, that has, as parts, all and only those parts of x that are disjoint from y. UC is a composition principle and a schema axiom stating that for each first-order formula, the minimal upper bound of those things satisfies that formula. I will return to these principles later. The following two main features of CM are remarkable: (1) CM is extensional as two objects cannot be identical without sharing the same proper parts; (2) CM does not have a bottom element, i.e. an element that is part of every other element. This demarcates the difference between CM and set theory. Interestingly, Tarski demonstrated that all algebraic models of second-order (or plural) CM, where UC ranges over sets of mereological entities, are complete Boolean algebras without the bottom element.

Furthermore, Cotnoir and Varzi provide examples of finite and infinite models of CM. Finite models, unlike infinite ones, always satisfy the atomicity thesis, stating that every object has atomic parts, which are parts without any further proper parts. Consequently, CM is neutral about the mereological final constitution. Ultimately, Cotnoir and Varzi also present alternative axiomatic systems in the formulation of CM, using different primitive notions such as proper part, disjunction, or overlap as well as different decomposition or composition axioms.

In Chapter 3, Cotnoir and Varzi discuss some potential counterexamples to the reflexivity/irreflexivity and antisymmetry/asymmetry of the relation of parthood/proper parthood, as well as the consequences of their failures. Additionally, Cotnoir and Varzi focus on counterexamples of transitivity, the failure of which allows for the existence of

mereological loops. In ordinary language, the relation of parthood is not always assumed to be transitive. For instance, if a doorknob is part of a door, and the door is part of a house, it is uncommon to consider the doorknob as part of the house. While some advocates of transitivity object to such counterexamples by saying that the abovementioned relation is not strictly mereological, others argue that the counterexamples depend on additional conditions tacitly imposed on parthood, like functionality, causality, or prescriptivity. However, this introduces new questions about how to define and handle such restrictive conditions that should be imposed upon the parthood relation. Nevertheless, answers to these questions do not fall within the scope of mereology, as they cannot be formulated by relying solely on the parthood relation and identity.

Additionally, Cotnoir and Varzi spotlight the mereological relation of "being an *immediate* part of" (x is an immediate part of y if and only if x is a proper part of y and there does not exist any proper part of y of which x is, in turn, a proper part), which is antitransitive. In the finite models of CM with more than one element, all entities have immediate parts. This is not yet true for the infinite models of CM due to the compatibility of CM with denseness (if x is a proper part of y, then there exists a z such that x is a proper part of z and z is a proper part of y). Further counterexamples to transitivity are introduced by invoking metaphysical contexts where, for instance, multilocation or time-travels are admitted. Ultimately, Cotnoir and Varzi explore the possibility of introducing a notion of transitivity acceptable both by advocates and opponents of transitivity.

In Chapter 4, Cotnoir and Varzi introduce decomposition principles other than R. Among the most debated, we find Strong Supplementation (SS) and Weak Supplementation (WS). SS states that if y is not part of x, then there exists a part of y completely disjoint from x. WS states that if x is a proper part of y, then there must exist another part of y that is completely disjoint from x. WS is motivated by the idea that a mereological entity cannot have only one proper part. Remarkably, SS follows from R, and WS follows from SS under the assumption that the relation of parthood is a partial order. Additionally, Cotnoir and Varzi discuss the relations between different types of WS depending on the formalization of the

relation of proper parthood one may adopt. For instance, one could define "x is a proper part of y" as holding if and only if x is part of y and x is not equal to y, or if and only if x is part of y but y is not part of x. Furthermore, Cotnoir and Varzi explore decomposition principles weaker than WS, such as the so-called Weak/Strong Company (if x is a proper part of y, then there exists a z that is a proper part of y and is neither identical to nor part of x) (SS implies WS), or quasi-supplementation (if x is a proper part of y, then y must have at least two proper parts, and they must be disjoint).

Ultimately, Cotnoir and Varzi discuss Atomism and its opposite, Atomlessness (nothing is an atom). CM supplemented with Atomlessness is a maximally consistent and decidable theory. The same goes for Atomism, if one adds either that the number of atoms is infinite or that it is exactly k, for a certain natural number k. Atomism and Atomlessness are incompatible, but can be reconceived if weakened. Weak Atomism requires that there is at least one atom, while Weak Atomlessness requires that there is at least one entity without atomic parts. CM supplemented with the last two principles and with the requests that the atoms are either infinite or exactly k constitutes a maximally consistent and decidable theory.

In Chapter 5, Cotnoir and Varzi delve into the principles of composition of which the upper-bound fusion is one of several possible options. Alternative definitions of fusion are possible, but, all else being equal, mereological theories that differ on the notion of fusion are not necessarily equivalent. Cotnoir and Varzi also explore the philosophical implications of the principles of composition. Among these, the Special Composition Question (SCQ) stands out. SCQ seeks to identify the necessary and sufficient criteria for a set, S, of entities to have a mereological fusion. There are two extreme answers to this question: universalism (S always has a fusion) and nihilism (S never has a fusion), as well as several moderate answers. The moderate answers suggest that fusion is allowed if the members of S satisfy a specific criterion, and the differences among these answers stem from the criteria adopted. UC is a non-full-blown universalist answer to SCQ as only countable infinite collections of entities have a fusion. UC also implies the existence of the universe, the entity of which every entity is part, against which one may argue. Additional

significant issues involve the uniqueness of fusion, the problem of composition as identity, and notions of fusion that do not adhere to commutativity, associativity, or idempotence.

In Chapter 6, the final chapter, Cotnoir and Varzi explore possible extensions or modifications of CM, conceived as a first-order logical theory equipped with the binary relation of parthood. Cotnoir and Varzi present such alternatives to CM following three main directions: (i) extensions to the second order of CM, specifically Lewis's megethology, which involves admitting plural quantification in CM; (ii) modifications to CM achieved by employing non-classical logics, such as trivalent, fuzzy, or paraconsistent logics; (iii) modifications aimed at modeling certain philosophical intuitions regarding time and modality, as well as the so-called mereological indeterminacy according to which entities' mereological structures is *de re* indeterminate.

Ultimately, *Mereology* stands as a highly significant academic achievement, addressing numerous formal and metaphysical aspects related to mereology. It lends itself to a continuous reading from the beginning until the end, yet it is equally approachable starting from various intermediate points for those who already have some familiarity with mereology. This accessibility makes this work an essential starting point for beginners and an indispensable reference for those who already have some acquaintance with the subject. Additionally, *Mereology* provides a comprehensive bibliography that encompasses most of the primary and secondary works published in this fascinating field of knowledge.

It is noteworthy, however, that the literature includes attempts to understand the relation of parthood that are not explicitly presented in *Mereology*. For example, Yablo (2015) defends that "x is part of y" if and only if x cannot change under certain aspects without y changing under the very same aspects. Indeed, Yablo introduces interpretations of the parthood relation that deviate from typical part-like characteristics, despite adhering to the commonly adopted logical characterization of the parthood relation. *Mereology* would have benefited from the inclusion of these accounts, as well as from a more in-depth discussion of their relations.

Bibliography

Franz Brentano, *Kategorienlehre*, Meiner, Leipzig 1933; *The Theory of Categories*, Alfred Kastil (ed.), Roderick M. Chisholm and Norbert Guterman (trans.), Nijhoff, The Hague 1981

Aaron Cotnoir, Achille Varzi, *Mereology*, Oxford University Press, Oxford 2021

Edmund Husserl, *Logische Untersuchungen*, Niemeyer, Halle, 1900-1901. 2nd edition: 1913 (vol. 1 and vol. 2:i) and 1921 (vol. 2:ii); *Logical Investigations*, John N. Findlay (trans.), Routledge & Kegan Paul, London 1970

Harry S. Leonard, Nelson Goodman, *The calculus of individuals and its uses*, in «The Journal of Symbolic Logic», 5, 1940, pp. 45-55

Stanisław Leśniewski, *Podstawy ogólnej teoryi mnogości.* Prace Polskiego Koła Naukowego w Moskwie, Moskow 1916; *Foundations of the general theory of sets*, David I. Barnett (trans.), in S. Leśniewski, *Collected Works*, vol. I, S.J. Surma *et al.* (eds.), Kluwer, Dordrecht 1992, pp. 174-382

Stephen Yablo, *Parts and differences*, in «Philosophical Studies», 173, 1/2016, pp. 141-157

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