Valentin Goranko
Temporal Logics
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Temporal Logics by Valentin Goranko is an overview and an introduction to the themes, problems and formal systems of temporal logic, the branch of logic dedicated to formalizing reasoning about both time itself and how states of affairs change over time. The book examines these topics to varying degrees of detail and technicality. It begins with the philosophical issues that gave rise to the discussions on temporal reasoning, then follows the development of temporal logic by presenting the modelling and logical tools which it offers. While describing the relevant logical systems, the book also takes the opportunity to explore their applications in various fields and their more technical meta-theoretic properties. The book assumes (and requires) familiarity with both classical logic and modal logic. For convenience, we also presuppose this background knowledge.

Chapter 1 offers a historical overview of the development of temporal logic. The chapter describes some selected classic problems and debates involving temporal reasoning. Among these, we find Zeno's paradoxes, Aristotle's "There will be a sea-battle tomorrow" example from his argument against assigning truth values to future contingents, Diodorus Cronus' Master Argument, which rejects the existence of a proposition that is possible but neither is nor will be true, and the medieval debates on determinism, free will and God's omniscience. The examples serve both as motivations for the field of temporal logic and as points of reference to illustrate systems in the later chapters.

Chapter 2 is divided into two main sections, each of which introduces a different foundation for formalizing time. The first describes instant-based models, where time is conceived as a set of instants endowed with a precedence relation (intuitively, an instant precedes another if the first is in the past compared to the second). Various properties that the precedence relation can pos-

sess (such as reflexivity, transitivity and linearity) are listed and briefly explained intuitively. The second section is dedicated to interval-based models of time, where the primitive entities for temporal reasoning are time intervals over linearly flowing time. In this section, we also find a complete description of all thirteen relations that can occur between two intervals: identity, having the same starting point, having the same ending point, one being included in the other, overlapping, one beginning at the end of the other, one being entirely after the other, and their respective converses.

In Chapter 3, we find a description of Prior's system of Tense Operators (TL), which interprets the classical propositional language extended with temporal operators *P*, *F*, *F* and (respectively, sometimes in the past, future, and always in the past, future) in an instant-based semantics. For instance, *P*φ intuitively means 'φ was true at some past instant'. After this introduction, the later sections cover how to translate the system into first order logic (along with a comparison of the two approaches) and a detailed exposition of the standard axiomatization of TL.

Chapter 4 outlines some logical tools particularly useful for dealing with linear time, i.e. over models that represent time as a sequence of instants linearly ordered by the precedence relation. The first two sections introduce, illustrate and axiomatize the operators *nexttime* (*X*) and *since/until* (*S*, *U*). *X*φ formalizes "φ is true at the immediate successor of this instant", φ*S*ψ formalizes "φ has held true since some past instant where ψ was true" and *U* is the converse of *S*. Using these operators, the later sections of the chapter describe the construction and axiomatization of the Linear Time Temporal Logic LTL, which is widely used in computer science to formalize infinite computations.

Chapter 5 introduces the notion of branching time, i.e. the idea that all instants have a fixed past, but some might be preceded by multiple instants that are not related by the precedence relation, allowing for non-linear temporal evolution. First, historical motivations for studying branching time are introduced. Then, two sections describe Prior's formalizations and critiques of both the Master Argument (introduced in Chapter 1) and Lavenham's Deterministic Argument (which argues that God's foreknowledge implies the determinism of the future). Prior's analyses motivate the development of two distinct branching time systems, which are explored in the later chapters. The chapter ends with an over-

view of tree-like models (sets of instants where instants might precede multiple other instants but the predecessors of each instant are linearly ordered) and of the key notions of history (a maximal linearly ordered set of instants in a tree) and bundle (a collection of histories whose union includes all instants of a tree).

Chapter 6 is dedicated to presenting and illustrating PBTL (Peircean Branching Time Logic), a system interpreting the language of TL in tree-like models, where *P* is interpreted as usual while Fφ is interpreted strongly as "φ is *necessarily* (i.e. in all histories) true at some point in the future". The evaluation of formulas is performed with respect to an instant and all histories including it, modelling the idea that there is no *actual* future. The final section covers Computation Tree Logic, an extension of PBTL which introduces strong versions of *X* and *U* to model infinite computations in a transition system.

Chapter 7 describes OBTL (Ockhamist Branching Time Logic). The language of OBTL includes standard versions of *P* and *F* and a modality ◊. Formulas are interpreted in a tree-based semantics with respect to an instant but, also, to a history passing through it, which takes the role of modelling the *actual* future. Intuitively, *F*φ means "φ will be true at some point in the *actual* future" and ◊φ means "φ may be true at some point in the future". The chapter illustrates OBTL's formal semantics and expressive power, then, it explores OBTL's relation with PBTL and discusses some possible axiomatizations. After a section describing the highly expressive Full Computation Tree Logic CTL*, we find mentions of various alternative systems for branching time and an illustration of their evaluation of Aristotle's "*There will be a sea-battle tomorrow*" example.

Chapter 8 tackles the problem of extending the scope of temporal logic to the first-order case. The first section outlines the main setup choices for the semantics. First, four ways in which the domain can change with time are described. Denoting the domain at an instant *t* by *D(t)*, the options are: 1. *D(t)* remains constant for any ; 2. Individuals are added to *D(t)* over time as they come into being 3. Individuals are gradually removed from *D(t)* as they cease being; 4. *D(t)* contains only those individuals that exist at the instant . Then, for quantification, two alternatives are discussed: the presentist view (quantifiers range over the local domain) and the eternalist view (quantifiers range over the union of all local domains). The subsequent sections are dedicated to First-Order

Temporal Logic (FOTL), a basic system to accommodate any of the above setup choices. First, the system is introduced. Then, after describing possible ways to assign variables and define semantic clauses in the system, illustrations and axiomatizations are presented for both a presentist and an eternalist version of FOTL, as well as a way to connect constant and varying domain semantics. The chapter ends with an application of the formal tools developed in the earlier sections to designations in natural language.

The first section of Chapter 9 introduces HS, a popular interval-based temporal logic. HS extends the language of classical propositional logic with twelve unary modal operators, each representing one of the twelve non-identity relations between the intervals of Chapter 2, and evaluates formulas with respect to intervals defined by pairs of ordered instants. The second section is an overview of various logics: logics with *nominals*, i.e. symbols associated with specific instants, metric logics, which can express the temporal distance between two instants, and real-time logics, combining the other two for instants on the real line. A third section describes extensions of the basic temporal logics: temporal logics of agency, useful to reason about agents, temporal-epistemic logics, which combine temporal reasoning with logics of knowledge, and spatial-temporal logics, tightly connected with physics and AI. In the last section, we find a summary of the applications of temporal logic to three fields: computer science, artificial intelligence and linguistic analysis of tense and discourse in natural language. The final sections of the book include an extensive list of references and a brief summary of the topics that are not presented.

As a whole, *Temporal Logics* is an accessible yet comprehensive overview of the field of temporal logic. As such, it suits a diverse audience. The book is a great first introduction to temporal logic for any reader with a basic background in classical and modal logic. At the same time, it can serve as an entry point for those who are only familiar with a specific branch of temporal logic to gain a wider perspective on the field. The extensive amount of well-organized references provided in each chapter and in the final section of the book also makes it a useful resource for those wishing to expand their knowledge of temporal logic by exploring the literature.

The content of the book is structured and organized very effectively. The discussion of philosophical motivations helps ground the technical exposition in a broader historical context, while the frequent references to earlier sections form a more general picture

of how various branches of temporal logics connect with each other. Additionally, the inclusion of many field-specific remarks and the numerous descriptions of applications and connections with other fields (such as philosophy, computer science, AI and linguistics) provide a broader context to the formal developments.

Stylistically, the presentation is very readable. The initial chapters and the sections where new systems are introduced favor examples and intuitions over technical detail. This choice makes the material accessible to readers unfamiliar with temporal logics. As a result, the more technical sections covering axiomatizations and validities of the systems also feel less dense, even when they go into further detail.

Overall, *Temporal Logics* provides an accessible and well-structured general view of the field of temporal logic. It is a valuable contribution to the field, both as an introductory work for newcomers and as a thorough guide for those looking to advance their studies.